The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 10, 2010 Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 3 hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- **Power off all cell phones**
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise. If you cite a reference, then please also provide the page number and quote you are using.**

Problem	Point Value	Your score	Topic
1	10		Mathematical Modeling
2	10		Differential Equation Rhythm
3	10		Differential Equation Blues
4	10		Discrete-Time Stability
5	10		System Identification
6	15		Discrete-Time Filter Analysis
7	15		Discrete-Time Filter Design
8	10		Sinusoidal Signal
9	10		Sinusoidal Amplitude Demodulation
Total	**100**		

Final Exam Problem 1. *Mathematical Modeling*. 10 points

Consider a signal $y(t)$ that is the continuous-time output of a light switch.

The signal $y(t)$ is one when the light switch is “on”, and zero when it is “off”.

(a) Sketch $y(t)$ when the light switch is “off” before time $t = 0$ and turns “on” at $t = 0$ and stays “on”. Give a mathematical definition of $y(t)$ in terms of the unit step function $u(t)$. Please give the value of $u(0)$ that you are using. 5 points.

(b) Sketch $y(t)$ when the light switch turns “on” at $t = 0$ s and turns “off “at $t = 1$ s. Give a mathematical definition of $y(t)$ in terms of the unit step function $u(t)$. Please give the value of $u(0)$ that you are using. 5 points.

Final Exam Problem 2. *Differential Equation Rhythm*. 10 points.

Consider a continuous-time system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=\frac{d^2}{dt^2}x(t)$$

for $t \geq 0^+$.

(a) What are the characteristic roots of the differential equation? 2 points.

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. 4 points.

(c) Find the zero-input response for the initial conditions $y(0^+) = 0$ and $y'(0^+) = 1$. 4 points.

Final Exam Problem 3. *Differential Equation Blues.* 10 points.

Consider a continuous-time linear time-invariant (LTI) system with input $x(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2 y(t) = \frac{d^2}{dt^2} x(t)$$

for $t \geq 0^-$.

(a) Give a formula for the transfer function in the Laplace domain. 2 points.

(b) Give the values of the poles and zeroes of the transfer function. 2 points.

(c) Give the region of convergence for the transfer function. 2 points.

(d) Give a formula for the frequency response of the LTI system. 2 points.

(e) What is the frequency selectivity of the LTI system? Lowpass, bandpass, bandstop, highpass, notch or all-pass? 2 points.

Final Exam Problem 4. *Discrete-Time Stability.* 10 points.

In this problem, the input signal is denoted by $x[n]$ and the output signal is denoted by $y[n]$. The input-output relationship of a linear time-invariant (LTI) system is defined as

$$y[n] - 1.6\, y[n-1] + K\, y[n-2] = x[n]$$

where K is an adjustable gain that can take any real value. By adjusting K, one can change the time response and frequency response of the system.

(a) What are the pole locations? Express your answer in terms of K. 3 points.

(b) For what values of K is the system bounded-input bounded-output stable? 2 points.

(c) Plot the pole locations in the z-domain as K varies. 2 points.

(d) Describe the frequency selectivity of the system (lowpass, highpass, bandpass, bandstop, notch, or allpass) for all possible values of K for which the system is bounded-input bounded-output stable. 3 points.

Final Exam Problem 5. *System Identification.* 10 points.

Consider a continuous-time system with input $x(t)$ and output $y(t)$.

You observe the following input-output relationships:

- When $x(t) = u(t)$, the output is $y(t) = u(t)$, assuming that $u(0) = 1$.
- When $x(t) = \cos(2 \pi t)$, the output is $y(t) = \frac{1}{2} + \frac{1}{2} \cos(4 \pi t)$.
- When $x(t) = \cos(4 \pi t)$, the output is $y(t) = \frac{1}{2} + \frac{1}{2} \cos(8 \pi t)$.

(a) Is the system linear and time-invariant? Please justify your answer. 5 points.

(b) Give a formula for the input-output relationship. 5 points.

Final Exam Problem 6. *Discrete-Time Filter Analysis*. 15 points.

A causal discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following difference equation:

$$y[n] = -0.8\, y[n-1] + x[n] - x[n-1]$$

(a) Draw the block diagram for this filter. 3 points.

(b) What are the initial conditions? What values should they be assigned? 3 points.

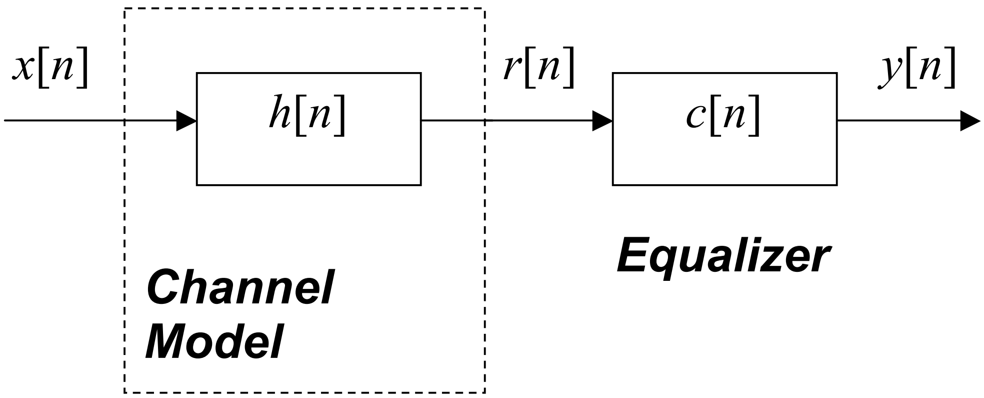
(c) Find the equation for the transfer function in the z-domain including the region of convergence. 3 points.

(d) Find the equation for the frequency response of the filter. 3 points.

(e) Describe the frequency selectivity of this filter as lowpass, bandpass, bandstop, highpass, notch, or allpass. Why? 3 points.

Final Exam Problem 7. *Discrete-Time Filter Design*. 15 points.

Consider the design of a discrete-time LTI filter with impulse response $c[n]$ to equalize a channel modeled as an LTI filter with impulse response $h[n]$:



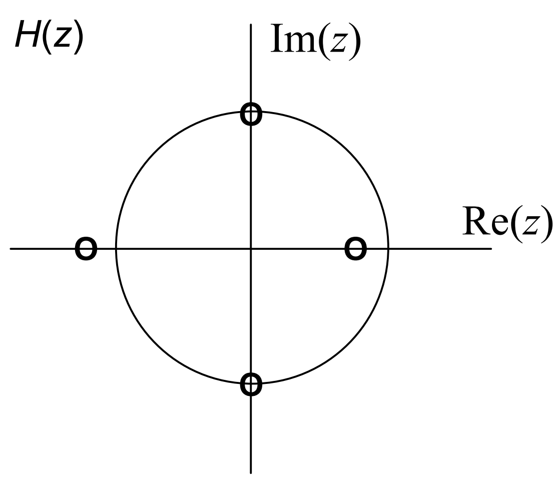
The channel model could represent a communication channel, a biomedical instrument or an audio system that distorts the input signal $x[n]$.

The equalizer is designed to compensate for the magnitude distortion in the channel as best it can. That is, the overall system from $x[n]$ to $y[n]$ should ideally have an all-pass response.

(a) Let $h[n] = 0.9^n\, u[n]$. What is the transfer function of the equalizer, $C(z)$? 4 points.

(b) Let $h[n] = \delta[n] - 2\,\delta[n-1]$. What is the transfer function of the equalizer, $C(z)$? 4 points.

(c) Let $H(z)$ have four zeros and no poles. The zeros are shown on the pole-zero diagram on the right. Place poles on the pole-zero diagram to design $C(z)$. You do not have to write the transfer function for $C(z)$. 7 points.



Final Exam Problem 8. *Sinusoidal Signal*. 10 points.

In practice, we cannot generate a two-sided sinusoid $\cos(2\,\pi\, f_c\, t)$, nor can we wait until the end of time to observe a one-sided sinusoid $\cos(2\,\pi\, f_c\, t)\, u(t)$.

In the lab, we can turn on a signal generator for a short time and observe the output in the time domain on an oscilloscope or in the frequency domain using a spectrum analyzer.

Consider a finite-duration cosine that is on for 1s given by the equation

$$c(t) = \cos(2\,\pi\, f_c\, t)\, \mathrm{rect}(t - \tfrac{1}{2})$$

where f_c is the carrier frequency (in Hz).

(a) Give a formula for the Fourier transform of $c(t)$. 3 points.

(b) Draw the magnitude of the Fourier transform of $c(t)$. 3 points.

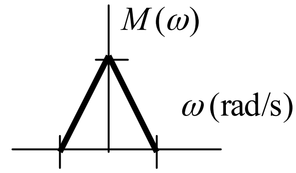
(c) Describe the differences between the magnitude of the Fourier transforms of $c(t)$ and a two-sided cosine of the same frequency. What is the bandwidth of each signal? 4 points.

Lowpass
Filter

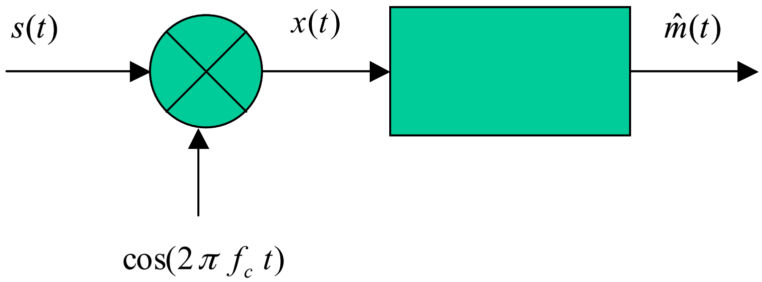
Final Exam Problem 9. *Sinusoidal Amplitude Demodulation*. $-\omega_m$ ω_m 10 points.

A lowpass, real-valued message signal $m(t)$ with bandwidth f_m (in Hz) is to be transmitted using sinusoidal amplitude modulation

$$s(t) = m(t) \cos(2 \pi f_c t)$$



where f_c is the carrier frequency (in Hz) and $f_c >> f_m$. The receiver processes the transmitted signal $s(t)$ to obtain an estimate of the message signal, $\hat{m}(t)$, as follows:



Hence, $x(t) = s(t) \cos(2 \pi f_c t)$. $M(\omega)$ is plotted above to the upper right.

(a) Plot the Fourier transform of $s(t)$, i.e. $S(\omega)$. 4 points.

(b) Plot the Fourier transform of $x(t)$, i.e. $X(\omega)$. 4 points.

(c) Give the smallest passband frequency and the largest stopband frequency for the lowpass filter to recover $m(t)$. 2 points.